

Generating entanglement among microwave photons and qubits in multiple cavities coupled by a superconducting qubit

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(Dated: July 12, 2011)

We propose a novel method for generating entangled coherent states of four microwave resonators (a.k.a. cavities) coupled by a superconducting qubit. We also show that a GHZ state of four superconducting qubits embedded in four different resonators can be created with this scheme. In principle, the proposed method can be extended to create an entangled coherent state of n resonators and to prepare a GHZ state of n qubits distributed over n cavities in a quantum network efficiently. More importantly, the qubit coupled cavities, with multiple qubits embedded in each cavity, can be used as the basic circuit block to build scalable quantum networks for quantum information processing.

PACS numbers: 03.67.Lx, 42.50.Dv, 85.25.Cp

Introduction. Recent progress in circuit cavity QED, in which superconducting qubits play the role of atoms in atom cavity QED, makes it stand out among the most promising candidates for implementing quantum information processing (QIP). Superconducting qubits, such as charge, flux, and phase qubits, and microwave resonators (a.k.a. cavities) can be fabricated using modern integrated circuit technology, their properties can be characterized and adjusted in situ, they have relatively long decoherence times [1], and various single and multiple qubits operations with high fidelity state readout have been demonstrated [2-6]. In particular, it has been demonstrated that a superconducting resonator provides a quantized cavity field which can mediate long-range and fast interaction between distant superconducting qubits [7-9]. Theoretically, it was predicted earlier that the strong coupling limit can readily be realized with superconducting charge qubits [10] or flux qubits [11]. Moreover, the strong coupling limit between the cavity field and superconducting qubits has been experimentally demonstrated [12,13]. All of these theoretical and experimental progresses make circuit cavity QED very attractive for QIP.

During the past decade, many theoretical proposals have been presented for the preparation of Fock states, coherent states, squeezed states, the Schrödinger Cat state, and an arbitrary superposition of Fock states of a single superconducting resonator [14-17]. Also, experimental creation of a Fock state and a superposition of Fock states of a single superconducting resonator using a superconducting qubit has been reported [18,19]. On the other hand, a large number of theoretical proposals have been presented for implementing quantum logical gates and generating quantum entanglement with two or more superconducting qubits placed in a cavity or coupled by a resonator (usually in the form of coplanar transmission line) [7,10,11,20]. Moreover, experimental demonstration of two-qubit gates and experimental preparation of three-qubit entanglement have been reported with superconducting qubits in a cavity [8,21,22]. However, realistic QIP will most likely need a large number of qubits and placing all of them in a single cavity quickly runs into many fundamental and practical problems such as the increase of cavity decay rate and decrease of qubit-cavity coupling strength.

Therefore, future QIP most likely will require quantum networks consisting of a large number of cavities each hosting and coupled to multiple qubits. In this type of architecture transfer and exchange of quantum information will not only occur among qubits in the same cavity but also between different cavities. Hence, attention must be paid to the preparation of quantum states of two or more superconducting resonators (hereafter we use the term cavity and resonator interchangeably), preparation of quantum states of superconducting qubits located in different cavities, and implementation of quantum logic gates on superconducting qubits distributed over different resonators in a network. It is known that all of these ingredients are essential to realizing large-scale quantum information processing based on circuit QED. Recently, Mariani *et al.* [23] have proposed a way for the manipulation and generation of nonclassical microwave field states as well as the creation of controlled multipartite entanglement with two resonators coupled by a superconducting qubit, and Strauch *et al.* [24] have proposed a method to synthesize an arbitrary quantum state of two superconducting resonators using a tunable superconducting qubit. Moreover, Wang *et al.* [25] have experimentally demonstrated the creation of an entangled NOON state of photons in two superconducting microwave resonators by using a superconducting phase qubit coupled to two resonators, and Mariani *et al.* have experimentally shuffled one- and two-photon Fock states between three resonators interconnected by two superconducting phase qubits [26]. These works opened a new avenue for building one-dimensional linear quantum networks of resonators and qubits [Fig. 1(a)].

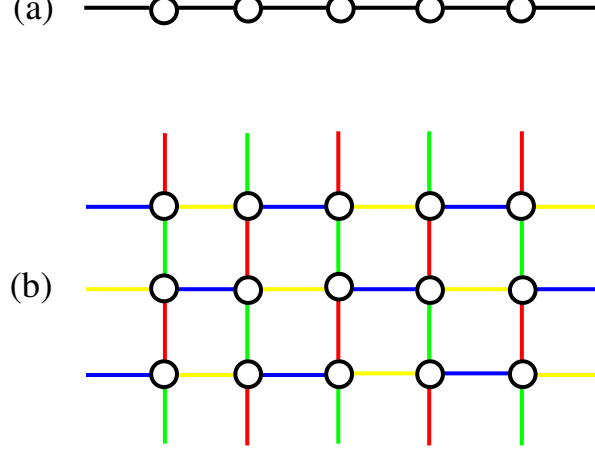


FIG. 1: (Color online) (a) One-dimensional linear network of resonators and qubits. (b) Two-dimensional linear network of resonators and qubits. In (a) and (b), a short line represents a resonator and a circle represents a superconducting qubit. In (a) a qubit is connected with two resonators, while in (b) a qubit is connected with four resonators. Note that the red, yellow, green and blue colors in (b) represent four different resonator frequencies, which are only needed regardless of the network size.

In this letter, we propose a way for generating entangled coherent states of four resonators using one three-level superconducting qubit as the inter-cavity coupler. This proposal operates essentially by bringing the transition between the two higher energy levels of the coupler qubit off-resonant with the resonator modes. In addition, we will show how to create a GHZ state of four superconducting qubits located in four different resonators using the coupler qubit. This work is fundamentally interesting because a quantum system composed of four resonators and a coupler qubit can serve as the basic building block to form extensive two-dimensional networks of resonators and qubits required for scalable QIP [Fig.1(b)].

As discussed below, our proposal has the following advantages: (i) Only one tunable superconducting qubit is needed; (ii) The operation procedure and the operation time are both independent of the number of resonators as well as the number of qubits in the cavities; (iii) No adjustment of the resonator mode frequencies is required during the entire operation; (iv) The entangled coherent states for the four resonators and the GHZ states of the four qubits in different cavities are generated deterministically; and (v) The proposed method can in principle be applied to create entangled coherent states of n resonators and to prepare a GHZ state of n qubits distributed over n cavities in a quantum network, for which the operational steps and the operation time do not increase as n becomes larger.

We stress that this proposal is quite general, and can be applied to other types of physical qubit systems with three levels, such as quantum dots and NV centers coupled to cavities.

Basic theory. Consider a three-level superconducting qubit A , with states $|0\rangle$, $|1\rangle$, and $|2\rangle$ as shown in Fig. 2, coupled to four resonators 1, 2, 3 and 4. Suppose that the relevant mode frequency of each resonator is off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition (*i.e.*, $\Delta_{c,i} = \omega_{21} - \omega_{c,i} \gg g_i$ for $i = 1, 2, 3, 4$) while decoupled from transitions between other levels of the qubit (Fig. 2). Here, g_i is the coupling constant between the resonator i and the $|1\rangle \leftrightarrow |2\rangle$ transition of qubit A . Under the condition that $\Delta_{c,2} - \Delta_{c,1}$, $\Delta_{c,3} - \Delta_{c,2}$, and $\Delta_{c,4} - \Delta_{c,3}$ are on the same order of magnitude as the coupling constants $g_1 \approx g_2 \approx g_3 \approx g_4$, the indirect interaction between any two resonators induced by qubit A is negligible. The effective interaction Hamiltonian of the entire system in the interaction picture is thus given by [27]

$$H_e = \hbar \sum_{i=1}^4 \frac{g_i^2}{\Delta_{c,i}} (|2\rangle_A \langle 2| - |1\rangle_A \langle 1|) a_i^\dagger a_i, \quad (1)$$

where the subscript A represents qubit A ; a_i and a_i^\dagger are the photon annihilation and creation operators of resonator i , respectively.

From this Hamiltonian, it is straightforward to see that if the resonator i is initially in a single-photon state $|1\rangle_{c,i}$, the time evolution of the state $|2\rangle_A |1\rangle_{c,i}$ of the system composed of qubit A and the resonator i is then given by

$$|1\rangle_A |1\rangle_{c,i} \rightarrow e^{ig_i^2 t / \Delta_{c,i}} |1\rangle_A |1\rangle_{c,i}, \quad (2)$$

which introduces a phase flip to the state $|1\rangle_A |1\rangle_{c,i}$ when the evolution time t satisfies $g_i^2 t / \Delta_{c,i} = \pi$. Note that the states $|0\rangle_A |0\rangle_{c,i}$, $|1\rangle_A |0\rangle_{c,i}$, and $|0\rangle_A |1\rangle_{c,i}$ remain unchanged under the Hamiltonian (1). In addition, based on the

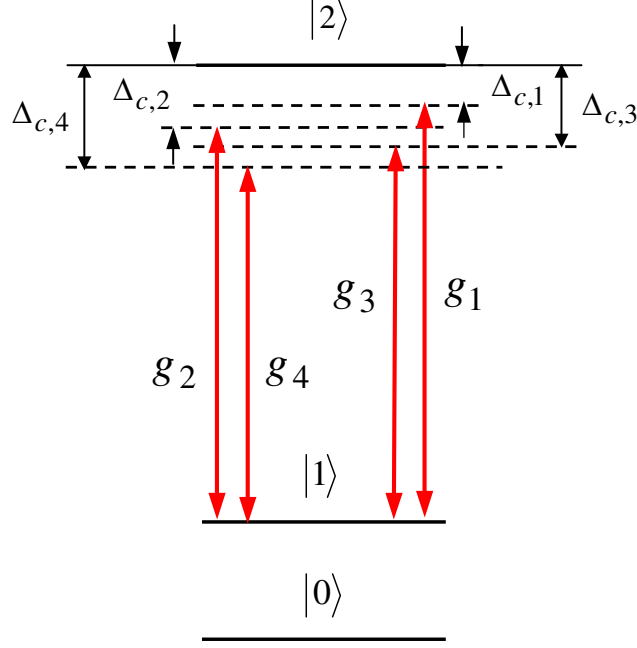


FIG. 2: (Color online) Qubit-cavity off resonant interaction for qubit A coupled to four resonators. $\Delta_{c,i} = \omega_{21} - \omega_{c,i}$ ($i=1,2,3,4$) is the detuning between the $|1\rangle \leftrightarrow |2\rangle$ transition frequency ω_{21} of the qubit A and the frequency $\omega_{c,i}$ of resonator i . The detunings $\Delta_{c,1}, \Delta_{c,2}, \Delta_{c,3}$, and $\Delta_{c,4}$ are set to be different to avoid the interaction between any two resonators. As shown here, the $|0\rangle \leftrightarrow |1\rangle$ level spacing is smaller than the $|1\rangle \leftrightarrow |2\rangle$ level spacing. Note that the level spacing between the two levels $|0\rangle$ and $|1\rangle$ can be larger than that between the two levels $|1\rangle$ and $|2\rangle$ because the level $|0\rangle$ is not used during the entire operation.

Hamiltonian (1), it is easy to see that if the resonator i is initially in a coherent state $|\alpha_i\rangle$, the time evolution of the state $|1\rangle_A |\alpha_i\rangle$ of the system composed of qubit A and the resonator i is then described by

$$|1\rangle_A |\alpha_i\rangle \rightarrow |1\rangle_A |\alpha_i \exp(ig_i^2 t / \Delta_{c,i})\rangle, \quad (3)$$

which leads to the coherent state of the i -th cavity evolve from $|\alpha_i\rangle$ to $|\alpha_i \exp(ig_i^2 t / \Delta_{c,i})\rangle$ when $g_i^2 t / \Delta_{c,i} = \pi$. Note that the state $|0\rangle_A |\alpha_i\rangle$ does not change under the Hamiltonian (1).

Creation of four-resonator entangled coherent states. Consider a system composed of four resonators and a superconducting qubit A [Fig. 3(a)]. The qubit A has three levels shown in Fig. 2. Initially, the qubit A is decoupled from all resonators, which can be realized by prior adjustment of the qubit level spacings [1,28,29]. The qubit A is initially prepared in the state $(|0\rangle_A + |1\rangle_A) / \sqrt{2}$ and each resonator is initially prepared in a coherent state [14,19], *i.e.*, $|\alpha_i\rangle$ for resonator i ($i = 1, 2, 3, 4$). To prepare the four resonators in an entangled coherent state, we now perform the following operations:

Step (i): Adjust the level spacings of the qubit A such that the field mode for each resonator is off resonant with the $|1\rangle \leftrightarrow |2\rangle$ transition (*i.e.*, $\Delta_{c,i} = \omega_{21} - \omega_{c,i} \gg g_i$ for resonator i) while far-off resonant with (decoupled from) the transition between other levels of the qubit A (Fig. 2). After an interaction time t , the initial state $(|0\rangle_A + |1\rangle_A) \prod_{i=1}^4 |\alpha_i\rangle$ of the whole system changes to (here and below a normalized factor is omitted for simplicity)

$$|0\rangle_A \prod_{i=1}^4 |\alpha_i\rangle + |1\rangle_A \prod_{i=1}^4 |\alpha_i \exp(ig_i^2 t / \Delta_{c,i})\rangle. \quad (4)$$

Both of the resonators and qubits can be fabricated to have appropriate resonator frequencies and qubit-cavity coupling strengths, such that $\frac{g_1^2}{\Delta_{c,1}} = \frac{g_2^2}{\Delta_{c,2}} = \frac{g_3^2}{\Delta_{c,3}} = \frac{g_4^2}{\Delta_{c,4}}$. Note that tunable qubit-cavity coupling strength has been proposed and demonstrated experimentally (e.g., [30-32]). For $g_i^2 t / \Delta_{c,i} = \pi$ ($i = 1, 2, 3, 4$), the system then evolves to

$$|0\rangle_A \prod_{i=1}^4 |\alpha_i\rangle + |1\rangle_A \prod_{i=1}^4 |-\alpha_i\rangle, \quad (5)$$

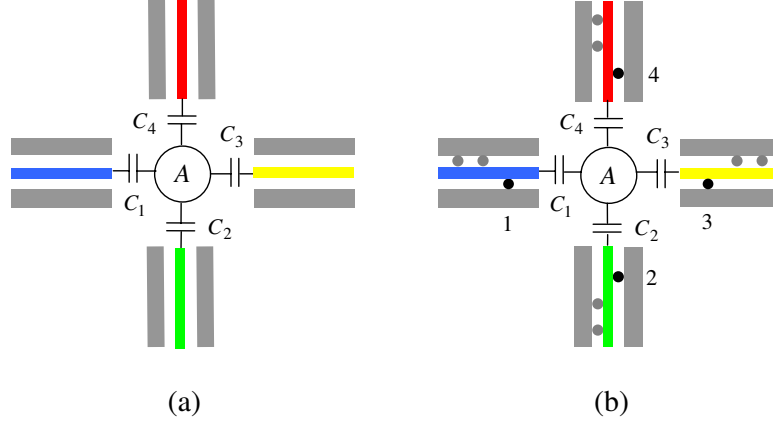


FIG. 3: (Color online) (a) and (b) Diagram of a superconducting qubit A (a circle at the center) coupled capacitively to four one-dimensional coplanar waveguide resonators through C_1, C_2, C_3, C_4 , respectively. In (b), a black or grey dot in each resonator represents a qubit. The four black-dot qubits (1, 2, 3, 4) are first prepared in a GHZ state, which can further be entangled with all other qubits (grey dots). For clarity, only three qubits in each cavity are shown.

according to Eq. (4). Here, $\langle \alpha_i | -\alpha_i \rangle = \exp(-2|\alpha_i|^2) \approx 0$ when α_i is large enough.

Step (ii): Adjust the level spacings of the qubit A such that it is decoupled (*i.e.*, far off-resonance) from all resonators. We then apply a classical $\pi/2$ -pulse (resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition of the qubit A) to transform the qubit state $|0\rangle_A$ to $|0\rangle_A + |1\rangle_A$ and $|1\rangle_A$ to $-|0\rangle_A + |1\rangle_A$. Thus, the state (5) becomes

$$|0\rangle_A \left(\prod_{i=1}^4 |\alpha_i\rangle - \prod_{i=1}^4 |-\alpha_i\rangle \right) + |1\rangle_A \left(\prod_{i=1}^4 |\alpha_i\rangle + \prod_{i=1}^4 |-\alpha_i\rangle \right). \quad (6)$$

We now perform a measurement on the states of the qubit A in the $\{|0\rangle, |1\rangle\}$ basis. If the qubit A is found in the state $|0\rangle$, it can be seen from Eq. (6) that the four resonators must be in the following entangled coherent state

$$\mathcal{N}_- (|\alpha_1\rangle |\alpha_2\rangle |\alpha_3\rangle |\alpha_4\rangle - |-\alpha_1\rangle |-\alpha_2\rangle |-\alpha_3\rangle |-\alpha_4\rangle), \quad (7)$$

Similarly, if the qubit is found in the state $|1\rangle$, then the four resonators must be in the following entangled coherent state

$$\mathcal{N}_+ (|\alpha_1\rangle |\alpha_2\rangle |\alpha_3\rangle |\alpha_4\rangle + |-\alpha_1\rangle |-\alpha_2\rangle |-\alpha_3\rangle |-\alpha_4\rangle), \quad (8)$$

where \mathcal{N}_{\mp} are the normalization factors. Thus, the entangled coherent states of the four resonators are generated deterministically.

It should be mentioned that for superconducting qubits, the level spacings can be readily adjusted by varying external control parameters (e.g., gate voltage and/or magnetic flux for the superconducting charge qubits, flux bias applied to the superconducting phase qubits and flux qubits, see e.g. [1, 28, 29]).

It is straightforward to show that by using a superconducting qubit coupled to n resonators (1, 2, ..., n) initially in the state $\prod_{i=1}^n |\alpha_i\rangle$, the n -resonator entangled coherent state $\prod_{i=1}^n |\alpha_i\rangle - \prod_{i=1}^n |-\alpha_i\rangle$ or $\prod_{i=1}^n |\alpha_i\rangle + \prod_{i=1}^n |-\alpha_i\rangle$ can be prepared by using the same procedure given above.

The entangled coherent states are an important resource in quantum information processing and quantum communication. In recent years, many schemes have been proposed for quantum computation and communication using coherent states [33-37]. The method proposed above allows one to prepare and manipulate coherent states distributed in a large number of cavities which is invaluable for scalable QIP.

Entangling qubits embedded in different cavities. Consider a system composed of four cavities coupled by a superconducting qubit A [Fig. 3(b)]. Each cavity hosts a two-level qubit 1, 2, 3, or 4, which is represented by a black dot [Fig. 3(b)]. The qubit A has a three-level structure as shown in Fig. 2. Each of the qubits (1, 2, 3, 4, A) is initially decoupled from the cavity/cavities and prepared in the state $|0\rangle + |1\rangle$. In addition, each cavity is initially in a vacuum state. The operations for preparing the qubits (1, 2, 3, 4) in a GHZ state are listed as follows:

Step (i): Adjust the level spacings of qubits (1, 2, 3, 4) such that the $|0\rangle \leftrightarrow |1\rangle$ transition of qubit i is resonant with the cavity i ($i = 1, 2, 3, 4$) for an interaction time $t = \pi/(2g)$. Without loss of generality, we assume that all qubit-cavity resonant coupling strengths are identical, i.e., $g'_i = g$ for all i . After the interaction time t , the initial states $\prod_{i=1}^4 (|0\rangle_i + |1\rangle_i) |0\rangle_{c,i}$ of the system changes to [38]

$$\prod_{i=1}^4 |0\rangle_i (|0\rangle_{c,i} - i |1\rangle_{c,i}). \quad (9)$$

Step (ii) Adjust the level spacings of qubits (1, 2, 3, 4) such that each of these qubits is decoupled from its own cavity, and adjust the level spacings of qubit A to bring the $|1\rangle \leftrightarrow |2\rangle$ transition of this qubit off resonant with the mode of each cavity (i.e., $\Delta_{c,i} = \omega_{21} - \omega_{c,i} \gg g_i$ for cavity i) while the transition between any other two levels of qubit A far-off resonant with (decoupled from) the mode of each cavity (Fig. 2). After an interaction time t , the initial state $\prod_{i=1}^4 |0\rangle_i (|0\rangle_{c,i} - i |1\rangle_{c,i}) (|0\rangle_A + |1\rangle_A)$ of the whole system changes to

$$\prod_{i=1}^4 |0\rangle_i \otimes \left[\prod_{i=1}^4 (|0\rangle_{c,i} - i |1\rangle_{c,i}) |0\rangle_A + \prod_{i=1}^4 (|0\rangle_{c,i} - i e^{ig_i^2 t / \Delta_{c,i}} |1\rangle_{c,i}) |1\rangle_A \right]. \quad (10)$$

With a choice of $\frac{g_1^2}{\Delta_{c,1}} = \frac{g_2^2}{\Delta_{c,2}} = \frac{g_3^2}{\Delta_{c,3}} = \frac{g_4^2}{\Delta_{c,4}}$ and for $g_i^2 t / \Delta_{c,i} = \pi$, we obtain from Eq. (10)

$$\prod_{i=1}^4 |0\rangle_i \otimes \left[\prod_{i=1}^4 (|0\rangle_{c,i} - i |1\rangle_{c,i}) |0\rangle_A + \prod_{i=1}^4 (|0\rangle_{c,i} + i |1\rangle_{c,i}) |1\rangle_A \right]. \quad (11)$$

Step (iii) Adjust the level spacings of qubit A such that this qubit is decoupled from each cavity. Then, adjust the level spacings of qubits (1, 2, 3, 4) such that the $|0\rangle \leftrightarrow |1\rangle$ transition of qubit i is resonant with the mode of cavity i ($i = 1, 2, 3, 4$) for an interaction time $t = \pi/(2g)$. After this step of operation, the state (11) becomes

$$\left[\prod_{i=1}^4 (|0\rangle_i - |1\rangle_i) |0\rangle_A + \prod_{i=1}^4 (|0\rangle_i + |1\rangle_i) |1\rangle_A \right] \otimes \prod_{i=1}^4 |0\rangle_{c,i}. \quad (12)$$

The result (12) shows that after the operation of this step, the qubit system is disentangled from the cavities but the qubits (1, 2, 3, 4) are entangled with qubit A .

Step (iv) Adjust the level spacings of the qubits (1, 2, 3, 4) such that these qubits are decoupled from their cavities. Then, apply a classical pulse (resonant with the $|0\rangle \leftrightarrow |1\rangle$ transition) to qubit A to transform the state $|0\rangle_A$ to $|0\rangle_A + |1\rangle_A$ while the state $|1\rangle_A$ to $-|0\rangle_A + |1\rangle_A$. Thus, the state (12) changes to

$$\left[|0\rangle_A \left(\prod_{i=1}^4 |-\rangle_i - \prod_{i=1}^4 |+\rangle_i \right) + |1\rangle_A \left(\prod_{i=1}^4 |-\rangle_i + \prod_{i=1}^4 |+\rangle_i \right) \right] \otimes \prod_{i=1}^4 |0\rangle_{c,i}, \quad (13)$$

where $|+\rangle_i = |0\rangle_i + |1\rangle_i$ and $|-\rangle_i = |0\rangle_i - |1\rangle_i$. We now perform a measurement on the states of qubit A in a basis $\{|0\rangle, |1\rangle\}$. If the qubit A is measured in the state $|0\rangle$, it can be seen from Eq. (13) that the qubits (1, 2, 3, 4) are prepared in the following entangled state

$$|-\rangle_1 |-\rangle_2 |-\rangle_3 |-\rangle_4 - |+\rangle_1 |+\rangle_2 |+\rangle_3 |+\rangle_4. \quad (14)$$

Since $|+\rangle_i$ is orthogonal to $|-\rangle_i$, the state (14) is a GHZ state of the four qubits (1, 2, 3, 4). On the other hand, when qubit A is measured in the state $|1\rangle$, it can be found from Eq. (13) that the qubits (1, 2, 3, 4) are prepared in the following entangled GHZ state

$$|-\rangle_1 |-\rangle_2 |-\rangle_3 |-\rangle_4 + |+\rangle_1 |+\rangle_2 |+\rangle_3 |+\rangle_4. \quad (15)$$

Hence, the GHZ states of the qubits (1, 2, 3, 4) are generated deterministically.

During the above GHZ-state preparation for the four qubits (1, 2, 3, 4), the other qubits in each cavity, which are represented by the grey dots in Fig. 3(b), are decoupled from the cavity mode by prior adjustment of their level spacings.

One can easily verify that in principle by using a superconducting qubit coupled to n cavities, n qubits (1, 2, ..., n) initially in the state $\prod_{i=1}^n |+\rangle_i$, which are respectively located in the different n cavities, can be prepared in an

entangled GHZ state $\prod_{i=1}^n |-\rangle_i - \prod_{i=1}^n |+\rangle_i$ or $\prod_{i=1}^n |-\rangle_i + \prod_{i=1}^n |+\rangle_i$, by using the same procedure described above. However, in practice since g_i is inversely proportional to n the number of cavities coupled to qubit A is limited to about ten to maintain sufficiently strong couplings.

Furthermore, based on the prepared GHZ state of n qubits $(1, 2, \dots, n)$, all other qubits (not entangled initially) in the cavities can be entangled with the GHZ-state qubits $(1, 2, \dots, n)$, through intra-cavity controlled-NOT (CNOT) operations on the qubits in each cavity by using the GHZ-state qubit in each cavity (i.e., qubit 1, 2, ..., or n) as the control while the other qubits as the targets. To see this clearly, let us consider Fig. 3(b), where the three qubits in cavity i ($i = 1, 2, 3, 4$) are the black-dot qubit i and the two grey-dot qubits, labeled as qubits $i2$ and $i3$ here. Suppose that the four black-dot qubits $(1, 2, 3, 4)$ (i.e., the GHZ-state qubits) were prepared in the GHZ state of Eq. (14), and each grey-dot qubit is initially in the state $|+\rangle$. By performing CNOT on various qubit pairs in each cavity, i.e., $C_{i,i2}$ and $C_{i,i3}$ on the qubit pairs $(i, i2)$ and $(i, i3)$ for cavity i , one can have all qubits in the four cavities (both black-dot and grey-dot qubits) prepared in a GHZ state $\prod_{i=1}^4 |-\rangle_i |-\rangle_{i2} |-\rangle_{i3} - \prod_{i=1}^4 |+\rangle_i |+\rangle_{i2} |+\rangle_{i3}$. Here, $C_{i,i2}$, defined in the basis $\{|+\rangle_i |+\rangle_{i2}, |-\rangle_i |+\rangle_{i2}, |+\rangle_i |-\rangle_{i2}, |-\rangle_i |-\rangle_{i2}\}$, represents a CNOT with qubit i (the GHZ-state qubit) as the control while qubit $i2$ as the target, which results in the transformation $|-\rangle_i |+\rangle_{i2} \rightarrow |-\rangle_i |-\rangle_{i2}$ while leaves the state $|+\rangle_i |+\rangle_{i2}$ unchanged. A similar definition applies to $C_{i,i3}$. Alternatively, using the prepared GHZ state of n qubits $(1, 2, \dots, n)$, one can have all other qubits in the cavities to be entangled with the GHZ-state qubits $(1, 2, \dots, n)$, by performing an intra-cavity multiqubit CNOT with the GHZ-state qubit (the control qubit) simultaneously controlling all other qubits (the target qubits) in each cavity [39].

Finally, it should be mentioned that quantum operations (e.g., quantum gates or information transfer) on/between two qubits, which are located at any places of the 2D quantum network [Fig. 1(b)], can be performed, by using the qubit-qubit coupling mediated through other qubits or cavities.

Discussion. Because $g_i^2/\Delta_{c,i} \ll g_i, \Omega$, where Ω is the Rabi frequency characterizing the strength of interaction between the classical microwave pulse and qubit A , the total time τ required for the creation of entangled coherent-state and GHZ-state can be approximated by

$$\tau \approx \pi \Delta_{c,i}/g_i^2, \quad i = 1, 2, 3, 4. \quad (16)$$

Here, $\Delta_{c,i}/g_i^2 = \Delta_{c,j}/g_j^2$ for $i \neq j \in \{1, 2, 3, 4\}$ (see the setting above) and τ much shorter than energy relaxation time T_1 and dephasing time T_2 of qubit A are assumed. For cavity i ($i = 1, 2, 3, 4$), the lifetime of the cavity mode is given by $T_{cav}^i = (Q_i/2\pi\nu_{c,i})/\bar{n}_i$, where Q_i and \bar{n}_i are the (loaded) quality factor and the average photon number of the cavity i , respectively. For four cavities, the lifetime of the cavity modes is given by $T_{cav} = \frac{1}{4} \min\{T_{cav}^1, T_{cav}^2, T_{cav}^3, T_{cav}^4\}$, which should be much longer than τ . Note that these conditions can be readily satisfied by superconducting qubits [1,40] and microwave resonators [41,42].

As an estimate, we choose $\Delta_{c,i} \sim 10g_i$ and assume $g_i/2\pi \sim 100$ MHz, which could be reached for a superconducting qubit coupled to a one-dimensional standing-wave CPW (coplanar waveguide) transmission line resonator [22]. For the value of g_i chosen here, we have $\tau \sim 50$ ns, much shorter than $\min\{T_1, T_2\} \sim 10 \mu\text{s}$ [40]. Thus, the effect of qubit decoherence caused by energy relaxation and dephasing can be made negligible by choosing parameters appropriately, for both entangled coherent-state creation and GHZ-state preparation.

Superconducting CPW transmission line resonators with a loaded quality factor $Q \sim 10^6$ have been experimentally demonstrated [41,42]. Without loss of generality, consider four resonators each with a center frequency ~ 5 GHz (e.g., Ref. [21]) and a quality factor $\sim 10^6$. From the discussion given above, it can be seen that each cavity was occupied by a single photon during the GHZ-state preparation. Thus, for the GHZ-state preparation we have $T_{cav} \sim 8 \mu\text{s} \gg \tau$.

A small coherent state with an average photon number varying from 0 to 10 can be created within present technique [19]. For the case when each cavity contains, on average, about three photons, we have $T_{cav} \sim 2.6 \mu\text{s}$ for the entangled coherent-state creation, which is much longer than τ . We remark that the T_{cav} here can be further increased by the use of cavities with improved quality factors.

For the choice of $\Delta_{c,i} \sim 10g_i$, it is estimated that the occupation probability for the level $|2\rangle$ of qubit A interacting with the cavity i (by nonresonant excitation) would be on the order of $4g_i^2/(4g_i^2 + \Delta_{c,i}^2) \sim 0.04$, which can be further reduced by increasing the ratio of $\Delta_{c,j}/g_j$.

In summary, we have proposed a novel method for creating four-resonator entangled coherent states and preparing a GHZ state of four qubits in four cavities, by using a superconducting qubit as the coupler. In principle, this proposal can be extended to create entangled coherent states of n resonators and to prepare GHZ states of n qubits distributed over n cavities in a network, with the same operational steps and the operation time as those of the four-resonator case described above. The proposed 2D quantum network of superconducting qubit coupled resonators each having multiple qubits embedded [see Fig. 1(b) and Fig. 3(b)] provides a more implementable architecture for scalable quantum information processing. This proposal has a distinct feature that only four different resonator frequencies are needed, regardless of the 2D network size. Finally, the proposed method is quite general, which can be applied to other types of qubits such as quantum dots and NV centers as long as the qubits can interact with cavities and have three levels.

C.P. Yang acknowledges funding support from the National Natural Science Foundation of China under Grant No. 11074062, the Zhejiang Natural Science Foundation under Grant No. Y6100098, and the funds from Hangzhou Normal University.

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